Title: More Pressure, More Work

Brief Overview:

This lesson will model the effect of changes in volume and pressure on the amount of work accomplished in a closed system and demonstrate an application of the fundamental theorem of calculus. Students will use the TI-83 calculator with the Calculator Based Laboratory (CBL) to:

(1) collect data from a simulation model.
(2) fit a prediction equation P(v) to the sample data.
(3) calculate subinterval estimates of area under the curve P(v).
(4) fit an equation to the area data W(v).
(5) determine the relationship between P(v) and W(v).

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  The students will demonstrate their knowledge and understanding of functions by generating a statistical data plot and finding an equation of best fit.

- **Mathematics as Communication**
  The students will discuss and conjecture procedures for fitting an equation to statistical data plots.

- **Mathematics as Reasoning**
  The students will test equations and develop criteria for determining a good fit.

- **Mathematical Connections**
  The students will recognize the relationship between physical applications and their mathematical representation. Calculus and physics will be inherently linked in this activity.

Grade/Level:

Grades 11-12

Duration/Length:

This activity will take 3 or 4 days. The activities may take longer than anticipated depending on class duration and student’s prior knowledge of CBL and differential/integral calculus.

Prerequisite Knowledge:

Students should have working knowledge and understanding of:

- Rectangular approximation method for calculating area under a curve
- The fundamental theorem of calculus
- Quantitative relationship between pressure, volume, and work in a closed system
- Working knowledge of the TI-83 graphing calculator and the CBL unit

Objectives:

Students will:

- work cooperatively in pairs.
- collect and organize data using the CBL equipment.
- organize, graph, and interpret a set of data using the graphing calculator as an assist.
- verify the fundamental Theorem of Calculus using physical relationship of pressure, volume, and work.

Materials/Resources/Printed Materials:

- Overhead projector and TI-83 with LCD panel
- TI-83 calculator
- CBL unit with pressure sensor attachment
- Student worksheets 1-3
- Pencils & paper

Development/Procedures:

- The teacher will explain the relationship between pressure and volume in a closed container and how their change results in work being done.
- Students will be given the objective and receive instructions on the conduct of this exploration exercise.
- Students work in pairs to do Activity #1, collect and plot pressure and volume data and find equation of best fit. (Determine type function and guess and test to find parameter values or use regression equations in calculator.)
- Students present findings to class using TI-83 with LCD panel and overhead projector.
- Students begin Activity #2, find the area of the subintervals under the pressure/volume curve, calculate the cumulative sum of these areas over the domain of the volume, and find an equation of best fit for this cumulative area data.
- Students will use the results of Activities #1 and #2 to demonstrate that the derivative of the cumulative area function W(v) is the function of the pressure/volume curve P(v).
- Students will complete Activity #3 individually as homework.

Evaluation:

- Group evaluation will be based on the accuracy of data collection and quality fit of equations to statistical data. Individual evaluation will be based on results obtained from exercise assigned for homework

Extension/Follow Up:

- Do similar activity using velocity vs time curve and demonstrate relationship of distance function to the velocity curve.

- Develop the concept of finding the volumes of solids by summing areas over an interval and demonstrating the derivative of the volume function is the area function.

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Introduction to Activities

Discussion: Under most circumstances when a gas in a closed cylinder is compressed its volume and pressure change. The volume and pressure act inversely in that as the volume expands or increases, the pressure decreases. Although both quantities can change, their product is a constant and can be expressed by the relationship

$$P = k/V \qquad \text{or} \qquad P*V = k \qquad \text{(k is positive)}$$

Also as the volume changes work is being accomplished since we are applying a force through a distance. This relationship can be represented by

$$W = F\bullet d \qquad \text{or} \qquad W = P(\Delta V) \qquad \text{where Pressure = Force/Area}$$
$$\text{Volume = Area * Distance}$$

In our activity, the cross-sectional area of the cylinder is constant and therefore a change in volume occurs with a change in d the distance the piston moves through the cylinder.

This exploration is broken into three parts:

1. Collecting of pressure-volume data and finding an equation of best fit.

2. Calculating the work, (W_i), accomplished for each change in volume, ΔV; developing a statistical plot of Work vs Volume; and finding an equation of best fit.

3. Comparing the equation obtained in Activity #1 with the derivative of the work equation in Activity #2 and developing some general conclusions based on the findings.

Form students into pairs to perform the explorations in Activities #1 and #2. Activity #3, which incorporates Activities #1 and #2, is designed to be completed individually as homework.

A key to Activity #3 is provided. The procedure and calculations necessary to accomplish Activity #2 are detailed in the Answer Key to Activity #3.

ACTIVITY # 1

Introduction:

In this activity you will use the CBL system and pressure sensor to investigate the general relationship between pressure and volume for air contained within a closed system.

Materials / Equipment:

1 CBL unit
1 TI-83 graphing calculator with unit-to-unit link cable.
1 Vernier Pressure Sensor with CBL-DIN adapter
1 Syringe attachment with short piece of plastic tubing
TI-83 programs PRESSURE ***** and RECTNGLS

Instructions:

The pressure in the syringe will be varied during the activity by decreasing the volume (pressing on the syringe plunger.

1. Attach the syringe to the pressure sensor with a short piece of tight-fitting plastic tubing.
2. Start the PRESSURE program on the TI-83 calculator.
3. Follow instructions on the TI-83 screen to collect data and construct a statistical plot. (Note: PRESSURE program sorts data in ascending order in lists L_1 and L_2.

Activity Data:

1. Record Data

Volume (L1)	Pressure (L2)

Table 1.1

2. Your data should show pressure decreasing as volume increases by the relationship $P = k / V$.

a. Find the equation of best fit by guessing and testing values for k.
   (1) To test this, enter the function $y = k/x$ into an unused function register. (Y_i)
   (2) Try different values of k by entering an estimate ($k = P*V$) at the homescreen, then storing it as the variable k. STO ALPHA K ENTER
   (3) Press graph to see data and curve together.
   (4) Continue this process until you find a value that gives a good fit.

K = _________

Modeling equation: ____________________

* Real-World Math with the CBL System, Programs for the TI-82 / TI-83

b. Find the equation of best fit using the appropriate regression program on the TI-83 calculator.

   (1) From the statistical plot, determine the type function that might best model the data.
   (2) Select the corresponding regression model: STAT select CALC from the menu, arrow down to PwrReg, press ENTER.
   (3) Type L1, L2, Y2 following PwrReg at the home screen. This computes the equation of best fit and places it in the Y2= register.
   (4) Press GRAPH to see data and equation together.

3. The modeling equation found using either method above can be used to predict syringe pressure values for given volumes.

a. To find this information, access the table set-up menu 2ND WINDOW and for the independent variable select <u>ASK</u> and press ENTER
b. Go to Table, 2ND GRAPH to access the table editor.
c. Enter the volumes as x-values and record the corresponding pressure in the table below.

Volume (cc) (x-values)	Pressure (atm) (y-values)
3.5	
15.7	
59	
100	
0.5	
0.03	

Table 1.2

Questions:

1. Could the volume ever be zero? Explain.

2. How would you describe the relationship between pressure and volume?

3. Under ideal conditions what would be the exponent value, x, for the volume in the equation $PV^x = k$?

ACTIVITY # 2

Introduction

In this activity the student will use the graphing calculator to investigate the relationship between the total work done and the change in volume for air contained within a closed syringe. The amount of work accomplished for each change in volume (ΔV = 2cc) during activity #1 is :

$W_i = P_i(\Delta V)$ where W_i = work done on interval i
P_i = pressure at interval i
ΔV= change in volume

Total work accomplished for the change in volume being

$W = \Sigma P_i \Delta V \approx \int f(V)dV$ where $P = f(V) = k/V$

Materials / Equipment:

1 TI 83 calculator
Data set from activity # 1
Program RECTNGLS (attached)

Instructions:

The area under the curve between the volume readings will be calculated in program RECTNGLS using the Midpoint Rectangular Approximation Method (MRAM).

1. Load RECTNGLS program into calculator using TI Graph Link.
2. Call up RECTNGLS while in the statistical plot of Activity #1 data (pressure vs volume data) to:
   a. Compute the work for each change in volume (2cc)
   b. Represent the area pictorially with rectangles for each change in volume
   c. Store the work done for each ΔV in list L3.
3. Compute the cumulative Work accomplished and store in list L4.
   a. STAT , select Edit, ENTER, arrow to top of column L4 then 2ND STAT , select OPS, select CumSum , 2ND 3 ENTER
   b. Develop a statistical plot of volume (L1 data) vs cumulative work (L4 data). 2ND Y= select Plot2, ENTER (edit plot parameters) then plot ZOOM select ZoomStat ENTER

Activity Data:

1. Record Data

Volume (L1)	Work vs ΔV (L3)	Total Work (L4)

Table 2.1

2. Your data should show amount of work done for each change in volume decreases as total volume increases or stated another way, the potential to do work is decreasing as the volume expands in this closed system.

3. Find the equation of best fit. (determine type function from data plot)
   STAT , select CALC, select LnReg, ENTER

4. LnReg appears on homescreen. Type in (L_1, L_4, Y_3) and hit ENTER . Check correlation of fit r^2.

5. The modeling equation found can be used to predict work accomplished, or required, for changes in the volume.

a. To find this information using the TI-83 calculator, access the table set-up menu. 2ND WINDOW and the independent variable and select ASK and press ENTER

b. Go to Table 2ND GRAPH to access the table editor.

c. Enter the volumes as x-values and record the corresponding work values (Y_3 values) in the table 2.2 below.

Volume x-values	Work (y-values)
7	
9	
20	
22	

Table 2.2

d. What do you notice about the amount of work accomplished or required to effect a constant change in volume for different initial volumes?

<u>Relationship Between the Curve $P = F(V)$ and $W = \Sigma f(V)\Delta V$:</u>

1. The fundamental theorem of calculus states that $d \int f(x)dx = f(x)$

2. Using the TI-83 find the derivative of the Work equation (Y_3) and compare it to the equation for Pressure.

a. Go to first open register under Y=. $\boxed{\text{Y=}}$, arrow down to $Y_4 =$

b. Enter nDeriv (Y_3,X,X). $\boxed{\text{MATH}}$, select nDeriv, $\boxed{\text{VARS}}$ select Y-vars, select Y3, then type (X, X)

c. Graph Y_1 and Y_4 on your statistical plot of Pressure vs Volume data.

3. Sketch the statistical plot and graphs below.

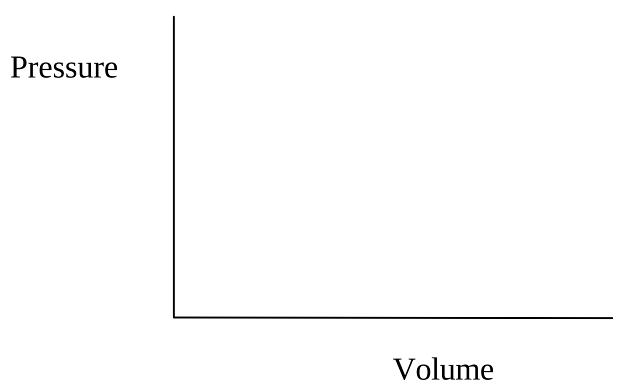


Figure 2.1

4. Note that the function $\frac{dW}{dV}$ is below (or to the left of) the data points. Try adjusting $\frac{dW}{dV}$ to the right one unit by editing Y_4 to read nDeriv (Y_3 , X, X-1). Explain why this is necessary.

<u>Questions:</u>

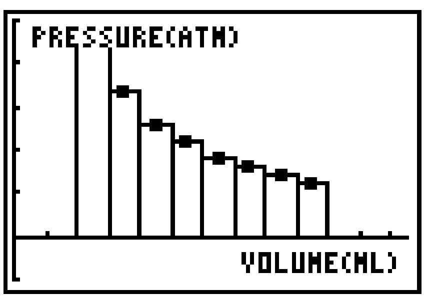
1. Does the volume have to change for work to be done?

2. What appears to be the relationship between the functions $P = f(V) = k/V$ and $W = \Sigma\ f(V)\Delta V = \Sigma P\Delta V$?

3. Based on your finding of the relationship between the pressure function and the work function, estimate the rate of change in the amount of work being accomplished at the instant the volume reaches 12 cc.

RECTNGLS Program

```
\start83\
\comment=Program file dated 06/25/98, 14:58
\name=RECTNGLS
\file=A:\RECTNGLS.TXT
0->Xmin
25->Xmax
2->Xscl
-1->Ymin
5->Ymax
1->Yscl
For(I,0,7,1)
Line(L1(I+1)+1,0,L1(I+1)+1,L2(I+1))
Line(L1(I+1)+1,L2(I+1),L1(I+1)-1,L2(I+1))
Line(L1(I+1)-1,L2(I+1),L1(I+1)-1,0)
End
2*L2->L3
Text(1,5,"PRESSURE(ATM)")
Text(55,55,"VOLUME(ML)")
\stop83\
```



ACTIVITY #3

Name ____________________________ Date_________________

In this activity you will use the model data (pressure vs volume) to :

- Graph a statistical plot
- Find an equation of best fit
- Find the equation that defines the work accomplished
- Estimate the work accomplished for a given volume change in the closed system
- Determine rate of change of work for specified volumes.

1. The following sample data was collected for changes in pressure and volume of a gas constrained in a closed tight circular cylinder.

Volume	Pressure
5	4.31
7	3.06
9	2.41
11	1.95
13	1.68
15	1.44
17	1.26
19	1.10

a. Plot the data to determine the type of function that best models these results.

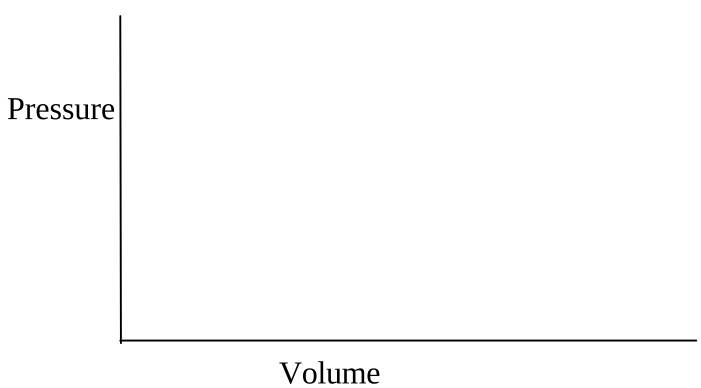


Figure 3.1

b. Find an equation of best fit.

k=________

p=________

b. Compute the amount of work accomplished, W_i, for each change in volume ΔV using MRAM or program Rectangles. Complete Data Table.

VOLUME	PRESSURE	W_i	TOTAL WORK

Table 3.1

2. Using the data in columns 1 and 3 in Table 3.1 determine the Total Work function.

a. Calculate Total Work accomplished at each interval of volume change and enter in list $\mathbf{L}_4$ and column 4 in Table 3.1.

$$W_{total} = \Sigma W_i$$

Calculator strokes:

$\boxed{\text{STAT}}$ (EDIT) Go to top margin of L_4

$\boxed{2^{nd}}$ $\boxed{\text{STAT}}$ OPS, CUMSUM, L_3, $\boxed{\text{ENTER}}$

b. Develop statistical plot of data (L_1,L_4) and determine type of function that best models these results.

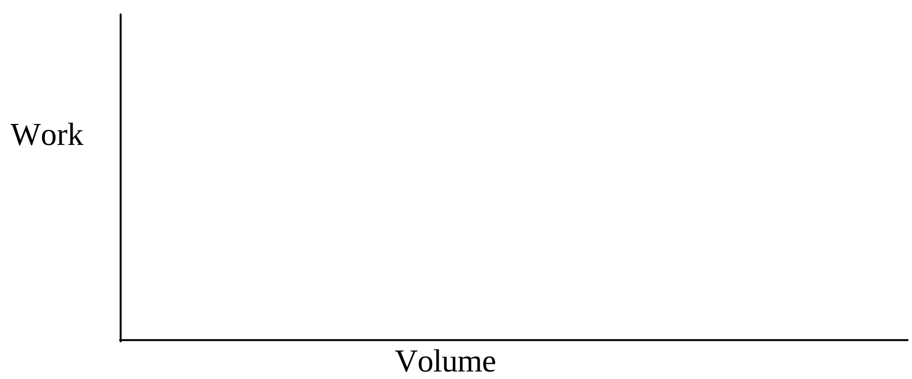


Figure 3.2

c. Find the equation of best fit: $W = \Sigma(P\Delta V)$, (Use LnReg under $\boxed{\text{STAT}}$ Calc)

W = ________________________ r^2 = _______________________

d. Using your equation found in 2c above:

(1) Predict the amount of work accomplished when the volume expands from 8 cc to 13 cc.

____

(2) Find $\frac{dW}{dV}$ of equation 2c. ______________________________

(3) Compare the rate of change of amount of work accomplished at 8cc and 13cc

rate of change of work = $\frac{dW}{dV} = P = \frac{k}{V}$

When V = 8cc $\frac{dW}{dV}$ = ______________

When V = 13cc $\frac{dW}{dV}$ = _______________

e. Graph $P = \frac{k}{V}$ found in 1b above and $\frac{dW}{dV}$ found in 2d(2) above on the statistical plot at 1a. (Figure 3.1 the pressure vs volume data)

<u>Questions:</u>

1. What is the relationship of Pressure and Work as a function of volume?

2. Find the integral of the function f(x) = $\frac{1}{x}$

3. If the velocity of an object is described by $v(t) = 2t^2$, how would you calculate the distance the object travels in the first three seconds after launch? Show your calculation.

ACTIVITY #3

Name **<u>Answer Key</u>** Date________________

In this activity you will use the model data (pressure vs volume) to:

Graph a statistical plot
Find an equation of best fit
Find the equation that defines the work accomplished
Estimate the work accomplished for a given volume change in the closed system
Determine rate of change of work for specified volumes.

1. The following sample data was collected for changes in pressure and volume of a gas constrained in a closed tight circular cylinder.

Volume	Pressure
5	4.31
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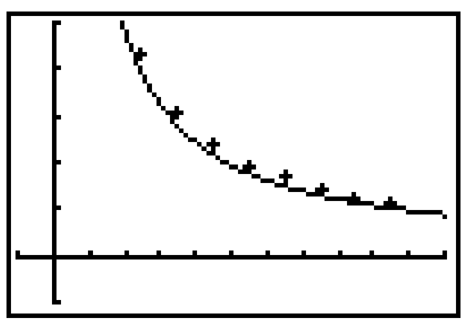
11	1.95
13	1.68
15	1.44
17	1.26
19	1.10

a. Plot the data to determine the type of function that best models these results.

Pressure



Volume

Figure 3.1

b. Find an equation of best fit.

k=__21.5______

$$p=\frac{21.5}{V}$$

c. Compute the amount of work accomplished, W_i, for each change in volume ΔV using MRAM or program Rectangles. Complete Data Table.

VOLUME	PRESSURE	W_i	TOTAL WORK
5	4.31	8.62	8.62
7	3.06	6.12	14.74
9	2.41	4.82	19.56
11	1.95	3.90	23.46
13	1.68	3.36	26.82
15	1.44	2.88	29.70
17	1.26	2.52	32.22
19	1.10	2.20	34.42

Table 3.1

2. Using the data in columns 1 and 3 in Table 3.1 determine the Total Work function.

a. Calculate Total Work accomplished at each interval of volume change and enter in list **L_4** and column 4 in Table 3.1.

$$W_{total} = \Sigma W_i$$

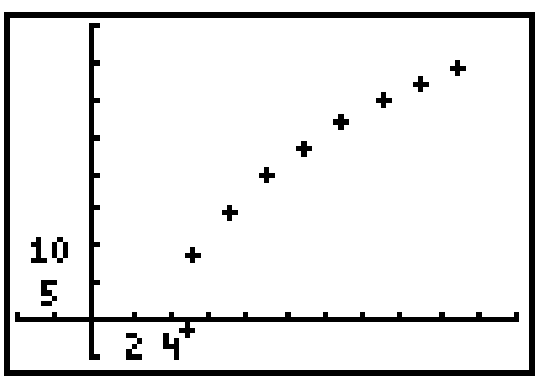
Calculator strokes:

$\boxed{\text{STAT}}$ (EDIT) Go to top margin of L_4

$\boxed{2^{nd}}$ $\boxed{\text{STAT}}$ OPS, CUMSUM, L_3, $\boxed{\text{ENTER}}$

b. Develop statistical plot of data (L_1,L_4) and determine type of function that best models these results.

Work



Volume

c. Find the equation of best fit: $W = \Sigma(P\Delta V)$ (Use LnReg under $\boxed{\text{STAT}}$ Calc)

W = -<u>22.9315 + 19.4255ln(V)</u> r^2 = <u>0.9996</u>

d. Using your equation found in 2c above:

(1) Predict the amount of work accomplished when the volume expands from 8 cc to 13 cc. 9.431

(2) Find $\frac{dW}{dV}$ of equation 2c. $\frac{19.4255}{V}$

(3) Compare the rate of change of amount of work accomplished at 8cc and 13cc

rate of change of work = $\frac{dW}{dV} = P = \frac{k}{(V-1)}$

Reminder: nDeriv was shifted to right 1 unit to account for discrete data calculated at midpoint of each interval. Replace V with (V-1).

When V = 8cc $\frac{dW}{dV} = \frac{19.4}{(8-1)} = 2.77$

When V = 13cc $\frac{dW}{dV} = \frac{19.4}{(13-1)} = 1.62$

d. Graph $P = \frac{k}{V}$ found in 1b above and $\frac{dW}{dV}$ found in 2d(2) above on the statistical plot at 1a. (Figure 3.1 the pressure vs volume data)

Questions

1. What is the relationship of Pressure and Work as a function of volume?

   The Pressure is the derivative of Work with respect to Volume

2. Find the integral of the function f(x) = $\frac{1}{x}$

   $$\int \frac{1}{x} dx = ln(x) + c$$

2. If the velocity of an object is described by v(t) = $2t^2$, how would you calculate the distance the object travels in the first three seconds after launch? Show your calculation.

   Find the area under the graph of v(t) $\int_0^3 2t^2 dt = 18$